

LINEAR PROGRAMMING

BASIC CONCEPTS AND FORMULA

Basic Concepts

1. Linear Programming

Linear programming is a mathematical technique for determining the optimal allocation of re- sources nd achieving the specified objective when there are alternative uses of the resources like money, manpower, materials, machines and other facilities.

2. Categories of the Linear Programming Problems

- i. General Linear Programming Problems.
- ii. Transportation Problems.
- iii. Assignment Problems.

3. Methods of Linear Programming

- i. Graphical Method
- ii. Simplex Method

4. Graphical Method

It involves the following:

- i. Formulating the linear programming problem
- ii. Plotting the capacity constraints on the graph paper.
- iii. Identifying feasible region and coordinates of corner points.
- iv. Testing the corner point which gives maximum profit.
- v. For decision – making purpose, sometimes, it is required to know whether optimal point leaves some resources unutilized.

5. Extreme Point Theorem

It states that an optimal solution to a LPP occurs at one of the vertices of the feasible region.

6. Basis theorem

It states that for a system of m equations in n variables (where $n > m$) has a solution in which at least $(n-m)$ of the variables have value of zero as a vertex. This solution is called a basic solution.

7. The Simplex Method

The simplex method is a computational procedure - an algorithm - for solving linear programming problems. It is an iterative optimizing technique.

8. The Simplex Method for Minimization and Maximization Problems

The simplex algorithm applies to both maximization and minimization problems. The only difference in the algorithm involves the selection of the incoming variable. In the maximization problem the incoming variable is the one with highest +ve net evaluation row (NER) element. Conversely, it is the most - ve variable that is selected as the incoming variable in a minimization problem. And if all elements in the NER are either positive or zero, it is the indication for the optimal solution.

9. Practical Application of Linear Programming

1. Industrial Application: To derive the optimal production and procurement plan for specific time period.
2. Administrative Application: in both academic circles and the area of business operations.

Question 1

A farm is engaged in breeding pigs. The pigs are fed on various products grown in the farm. In view of the need to ensure certain nutrient constituents (call them X, Y and Z), it becomes necessary to buy two additional products say, A and B. One unit of product A contains 36 units of X, 3 units of Y and 20 units of Z. One unit of product B contains 6 units of X, 12 units of Y and 10 units of Z. The minimum requirement of X, Y and Z is 108 units, 36 units and 100 units respectively. Product A costs Rs.20 per unit and product B Rs.40 per unit.

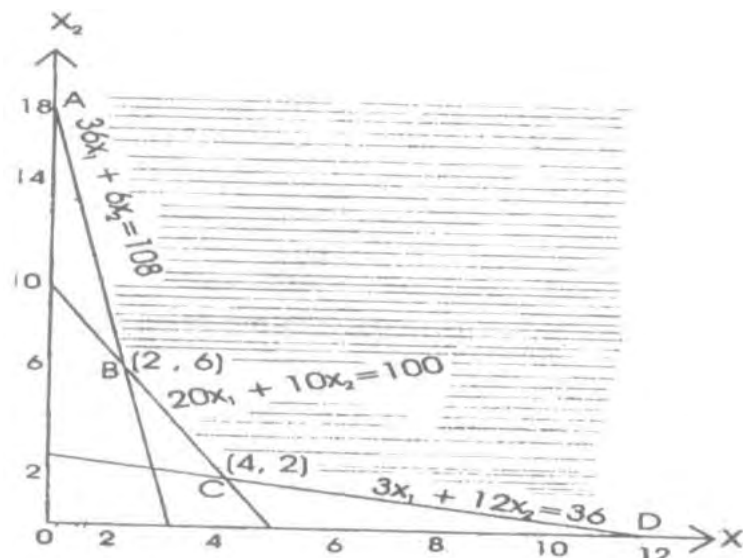
Formulate the above as a linear programming problem to minimize the total cost and solve this problem by using graphic method.

Answer

The data of the given problem can be summarized as under:

Nutrient constituents	Nutrient content in product		Minimum requirement of nutrient
	A	B	
X	36	06	108
Y	03	12	36
Z	20	10	100
Cost of product	Rs.20	Rs.40	

Let x_1 units of product A and x_2 units of product B are purchased. Making use of the above table, the required mathematical formulation of L.P. problem is as given below:



Minimize $Z = 20x_1 + 40x_2$ subject to the constraints

$$36x_1 + 6x_2 \geq 108$$

$$3x_1 + 12x_2 \geq 36$$

$$20x_1 + 10x_2 \geq 100$$

$$\text{and } x_1, x_2 \geq 0$$

For solving the above problem graphically, consider a set of rectangular axis x_1Ox_2 in the plane. As each point has the coordinates of type (x_1, x_2) , any point satisfying the conditions $x_1 \geq 0$ and $x_2 \geq 0$ lies in the first quadrant only.

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The constraints of the given problem as described earlier are plotted by treating them as equations:

$$36x_1 + 6x_2 = 108$$

$$3x_1 + 12x_2 = 36$$

$$20x_1 + 10x_2 = 100$$

Or

$$\frac{x_1}{2} + \frac{x_2}{18} = 1$$

$$\frac{x_1}{12} + \frac{x_2}{3} = 1$$

$$\frac{x_1}{5} + \frac{x_2}{10} = 1$$

The area beyond these lines represents the feasible region in respect of these constraints, any point on the straight lines or in the region above these lines would satisfy the constraints. The coordinates of the extreme points of the feasible region are given by

A = (0,18), B = (2,6), C = (4,2) and D = (12,0)

The value of the objective function at each of these points can be evaluated as follows:

Extreme Point	(x ₁ , x ₂)	Z = 20x ₁ + 40x ₂	
A	(0,18)	720	
B	(2,6)	280	
C	(4,2)	160	Minimum ←
D	(12,0)	240	

The value of the objective function is minimum at the point C (4,2).

Hence, the optimum solution is to purchase 4 units of product A and 2 units of product B in order to have minimum cost of Rs.160.

Question 2

A Computer Company produces three types of models, which are first required to be machined and then assembled. The time (in hours) for these operations for each model is given below:

Linear Programming

Model	Machine Time	Assembly Time
P III	20	5
P II	15	4
Celeron	12	3

The total available machine time and assembly time are 1,000 hours and 1,500 hours respectively. The selling price and other variable costs for three models are:

	P III	P II	Celeron
Selling Price (Rs.)	3,000	5,000	15,000
Labour, Material and other Variable Costs (Rs.)	2,000	4,000	8,000

The company has taken a loan of Rs.50,000 from a Nationalised Bank, which is required to be repaid on 1.4.2001. In addition, the company has borrowed Rs.1,00,000 from XYZ Cooperative Bank. However, this bank has given its consent to renew the loan.

The balance sheet of the company as on 31.3.2001 is as follows:

Liabilities	Rs.	Assets	Rs.
Equity Share Capital	1,00,000	Land	80,000
Capital reserve	20,000	Buildings	50,000
Profit & Loss Account	30,000	Plant & Machinery	1,00,000
Long-term Loan	2,00,000	Furniture etc.	20,000
Loan from XYZ Cooperative Bank	1,00,000	Cash	2,10,000
Loan from Nationalized Bank	50,000		
Total	5,00,000	Total	5,00,000

The company is required to pay a sum of Rs.15,000 towards the salary. Interest on long-term loan is to be paid every month@ 18% per annum. Interest on loan from XYZ Cooperative and Nationalised Banks may be taken as Rs.1,500 per month. The company has already promised to deliver three P III, Two P II and five Celeron type of computers to M/s. ABC Ltd. next month. The level of operation I the company is subject to the availability of cash next month.

The Company Manager is willing to know that how many units of each model must be manufactured next month, so as to maximize the profit.

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Formulate a linear programming problem for the above.

Answer

Let X_1 , X_2 and X_3 denote the number of P III, P II and Celeron computers respectively to be manufactured in the company. The following data is given:

	P III	P II	Celeron
Selling price per unit (Rs.)	3,000	5,000	15,000
Labour Material & other Variable cost per unit (Rs.)	2,000	4,000	8,000
Profit per unit (Rs.)	1,000	1,000	7,000

Since the company wants to maximize the profit, hence the objective function is given by:

$$\text{Maximize } Z = 1,000X_1 + 1,000X_2 + 7,000X_3 - (\text{Rs.}15,000 + 3,000 + \text{Rs.}1,500)$$

From the data given for time required for various models and the total number of hours available for machine time and assembly time, we get the following constraints:

$$20X_1 + 15X_2 + 12X_3 \leq 1,000 \text{ (Machine Time Restriction)}$$

$$5X_1 + 4X_2 + 3X_3 \leq 1,500 \text{ (Assembly Time Restriction)}$$

The level of operations in the company is subject to availability of cash next month i.e.; the cash required for manufacturing various models should not exceed the cash available for the next month.

The cash requirements for X_1 units of P III, X_2 units of P II and X_3 units of Celeron computers are:

$$2,000 X_1 + 4,000 X_2 + 8,000 X_3 \quad \dots\dots(1)$$

The cash availability for the next month from the balance sheet is as below:

Cash availability (Rs.)	=	Cash balance (Rs.2,10,000)
		- Loan to repay to Nationalized Bank (Rs.50,000)
		- Interest on loan from XYZ Cooperative bank and Nationalized bank (Rs.1,500)
		- Interest on long term loans $\left(\frac{0.18 \times 2,00,000}{12} \right)$
		- Salary to staff (Rs.15,000)

Linear Programming

$$\begin{aligned}\text{or, Cash availability} &= \text{Rs. } 2,10,000 - (\text{Rs. } 50,000 + \text{Rs. } 1,500 + \text{Rs. } 3,000 + 15,000) \\ &= \text{Rs. } 1,40,500 \quad \dots\dots(2)\end{aligned}$$

Thus, from (1) and (2),

$$2,000 X_1 + 4,000 X_2 + 8,000 X_3 \leq \text{Rs. } 1,40,500$$

The company has also promised to deliver 3 P III, 2 P II and 5 Celeron computers to M/s Kingspen Ltd.

$$\text{Hence, } X_1 \geq 3, X_2 \geq 2, X_3 \geq 5$$

The LP formulation of the given problem is as follows:

$$\text{Maximize } Z - 1,000 X_1 + 1,000 X_2 + 7,000 X_3 - (\text{Rs. } 15,000 + \text{Rs. } 3,000 + \text{Rs. } 1,500)$$

Subject to the constraints:

$$20 X_1 + 15 X_2 + 12 X_3 \leq 1,000$$

$$5 X_1 + 4 X_2 + 3 X_3 \leq 1,500$$

$$2,000 X_1 + 4,000 X_2 + 8,000 X_3 \leq \text{Rs. } 1,40,500$$

$$X_1 \geq 3, X_2 \geq 2, X_3 \geq 5$$

X_1, X_2 and X_3 can take only positive integral values.

Question 3

Computer Company produces three types of models, which are first required to be machined and then assembled. The time (in hours) for these operations for each model is given below:

Model	Machine Time	Assembly Time
P III	20	5
P II	15	4
Celeron	12	3

The total available machine time and assembly time are 1,000 hours and 1,500 hours respectively. The selling price and other variable costs for three models are:

	P III	P II	Celeron
Selling Price (Rs.)	3,000	5,000	15,000
Labour, Material and other Variable Costs (Rs.)	2,000	4,000	8,000

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The company has taken a loan of Rs.50,000 from a Nationalised Bank, which is required to be repaid on 1.4.2001. In addition, the company has borrowed Rs.1,00,000 from XYZ Cooperative Bank. However, this bank has given its consent to renew the loan.

The balance sheet of the company as on 31.3.2001 is as follows:

Liabilities	Rs.	Assets	Rs.
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Capital reserve	20,000	Buildings	50,000
Profit & Loss Account	30,000	Plant & Machinery	1,00,000
Long-term Loan	2,00,000	Furniture etc.	20,000
Loan from XYZ Cooperative Bank	1,00,000	Vehicles	40,000
Loan from National Bank	50,000		
Total	5,00,000	Total	5,00,000

The company is required to pay a sum of Rs.15,000 towards the salary. Interest on long-term loan is to be paid every month @ 18% per annum. Interest on loan from XYZ Cooperative Bank and Nationalised Bank may be taken as Rs.1,500 per month. The company has already promised to deliver three P III. Two P II and five Celeron Type of Computer of M/s ABC Ltd. next month. The level of operation in the company is subject to the availability of cash next month.

The Company Manager is willing to know that how many units of each model must be manufactured next month, so as to maximize the profit.

Formulate the linear programming problem for the above.

Answer

Let x_1 , x_2 and x_3 denote the number of P III, P II and Celeron Computers respectively to the manufactured in the company. The following data is given:

	P III	P II	Celeron
Selling Price per unit (Rs.)	3,000	5,000	15,000
Labour, Material and other Variable Costs p.u. (Rs.)	2,000	4,000	8,000
Profit per unit (Rs.)	1,000	1,000	7,000

From the data given for time required for various models and the total number of hours available for machine time and assembly time, we get the following constraints:

$$20x_1 + 15x_2 + 12x_3 \leq 1,000 \text{ (Machine Time Restriction)}$$

$$5x_1 + 4x_2 + 3x_3 \leq 1,500 \text{ (Assembly Time Restriction)}$$

The level of operations in the company is subject to availability of cash next month i.e.; the cash required for manufacturing various models should not exceed the cash available for the next month.

The cash requirements for x_1 units of P III, x_2 units of P II and x_3 units of Celeron computers are:

$$2,000x_1 + 4,000x_2 + 8,000x_3 \quad \dots\dots (1)$$

The cash availability for the next month from the balance sheet is as below:

Cash availability (Rs.) Cash balance (Rs. 2,10,000)

Loan to repay to Nationalized bank (Rs. 50,000)

Interest on loan from XYZ cooperative bank and Nationalized bank (Rs. 1500)

$$\text{Interest on long term loans} \left[\frac{0.18 \times 2,00,000}{12} \right]$$

Salary to staff (Rs. 15,000)

$$\begin{aligned} \text{Or, Cash availability} &= \text{Rs. } 2,10,000 - (\text{Rs. } 50,000 + \text{Rs. } 1,500 + \text{Rs. } 3,000 + \text{Rs. } 15,000) \\ &= \text{Rs. } 1,40,500 \quad \dots\dots (2) \end{aligned}$$

Thus, from (1) and (2),

$$2000X_1 + 4000X_2 + X_3 < \text{Rs. } 1,40,500$$

The company has also promised to deliver 3 P III, 2 P II and 5 Celeron computers to M/s. Kingspen Ltd.

$$\text{Hence, } X_1 > 3, X_2 > 2, X_3 > 5$$

Since the company wants to maximize the profit, hence the objective function is given by: Maximize $Z = 1000X_1 + 1000X_2 + 7000X_3$ - (Rs. 15000 + Rs. 3000 + Rs. 1500) The LP formulation of the given problem is as follow:

Maximize $Z = 1000X_1 + 1000X_2 + 7000X_3$ - (Rs. 15000 + Rs. 15000) Subject to the constraints:

$$20X_1 + 15X_2 + 12X_3 < 1000$$

$$5X_1 + 4X_2 + 3X_3 < 1500$$

$$2000X_1 + 4000X_2 + 8000X_3 < \text{Rs. } 1,40,500$$

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$$X_1 > 3, X_2 > 2, X_3 > 5$$

X_1, X_2 and X_3 can take only positive integral values.

Question 4

A manufacturing company produces two types of product the SUPER and REGULAR. Resource requirements for production are given below in the table. There are 1,600 hours of assembly worker hours available per week. 700 hours of paint time and 300 hours of inspection time. Regular customers bill demand at least 150 units of the REGULAR type and 90 units of the SUPER type. (8 Marks)

Table

Product	Profit/contribution Rs.	Assembly time Hrs.	Paint time Hrs.	Inspection time Hrs.
REGULAR	50	1.2	0.8	0.2
SUPER	75	1.6	0.9	0.2

Formulate and solve the given Linear programming problem to determine product mix on a weekly basis.

Answer

Let x_1 and x_2 denote the number of units produced per week of the product 'REGULAR' and 'SUPER' respectively.

$$\text{Maximise } Z = 50x_1 + 75x_2$$

Subject to

$$1.2x_1 + 1.6x_2 \leq 1,600 \text{ or } 12x_1 + 16x_2 \leq 16,000 \quad \text{-(i)}$$

$$0.8x_1 + 0.9x_2 \leq 700 \text{ or } 8x_1 + 9x_2 \leq 7,000 \quad \text{-(ii)}$$

$$0.2x_1 + 0.2x_2 \leq 300 \text{ or } 2x_1 + 2x_2 \leq 3,000 \quad \text{-(iii)}$$

$$x_1 \geq 150 \quad \text{-(iv)}$$

$$x_2 \geq 90 \quad \text{-(v)}$$

Let

$$x_1 = y_1 + 150$$

$$x_2 = y_2 + 90 \text{ where } y_1, y_2 \geq 0$$

$$\text{Maximize } Z = 50(y_1 + 150) + 75(y_2 + 90) \text{ or } Z = 50y_1 + 75y_2 + 14,250$$

Subject to:

$$12(y_1 + 150) + 16(y_2 + 90) \leq 16,000$$

$$8(y_1 + 150) + 9(y_2 + 90) \leq 7,000$$

$$2(y_1 + 150) + 2(y_2 + 90) \leq 3,000$$

$$\text{and } y_1, y_2 \geq 0$$

Adding slack variables s_1, s_2, s_3 , we get

Maximize $Z = 50y_1 + 75y_2 + 14,250$ subject to

$$12y_1 + 16y_2 + s_1 = 12,760$$

$$8y_1 + 9y_2 + s_2 = 4,990$$

$$2y_1 + 2y_2 + s_3 = 2,520$$

Table I

C_j			50	75	0	0	0	
C_b			y_1	y_2	s_1	s_2	s_3	
0	s_1	12,760	12	16	1	0	0	12760/16
0	s_2	4,990	8	9	0	1	0	4990/9
0	s_3	2,520	2	2	0	0	1	2520/2
Δ_j			-50	-75	0	0	0	

Table II

C_j			50	75	0	0	0
C_b			y_1	y_2	s_1	s_2	s_3
0	s_1	3889	-20/9	0	1	-16/9	0
75	y_2	554.44	8/9	1	0	1/9	0
0	s_3	1411	2/9	0	0	-2/9	1
Δ_j			50/3	0	0	75/9	0

Since all the elements in the index row are either positive or equal to zero, table II gives an optimum solution which is $y_1 = 0$ and $y_2 = 554.44$

Substituting these values we get

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$$x_1 = 0 + 150 = 150$$

$$x_2 = 90 + 554.44 = 644.44 \text{ and the value of objective function is}$$

$$Z = 50 \times 150 + 75 \times 644.44$$

$$= \text{Rs. } 55,833$$

Question 5

A company manufactures two products A and B, involving three departments – Machining, Fabrication and Assembly. The process time, profit/unit and total capacity of each department is given in the following table:

	Machining (Hours)	Fabrication (Hours)	Assembly (Hours)	Profit (Rs).
A	1	5	3	80
B	2	4	1	100
Capacity	720	1,800	900	

Set up Linear Programming Problem to maximise profit. What will be the product Mix at Maximum profit level ?

Answer

$$\begin{aligned} \text{Maximize } z &= 80x + 100y \text{ subject to} & x + 2y &\leq 720 \\ & & 5x + 4y &\leq 1800 \\ & & 3x + y &\leq 900 \\ & & x \geq 0, y &\geq 0 \\ \text{where} & & x &= \text{No. of units of A} \\ & & y &= \text{No. of units of B} \end{aligned}$$

By the addition of slack variables s_1 , s_2 and s_3 the inequalities can be converted into equations. The problems thus become

$$\begin{aligned} z &= 80x + 100y \text{ subject to} & x + 2y + s_1 &= 720 \\ & & 5x + 4y + s_2 &= 1800 \\ & & 3x + y + s_3 &= 900 \\ \text{and } x &\geq 0, & y \geq 0, s_1 \geq 0, s_2 \geq 0, s_3 &\geq 0 \end{aligned}$$

Table 1:

			80	100	0	0	0	
	Profit/unit	Qty.	X	Y	S ₁	S ₂	S ₃	
S ₁	0	720	I	2	1	0	0	$\frac{720}{2} = 360$
S ₂	0	1800	5	4	0	1	0	$1800/4 = 450$
S ₃	0	900	3	I	0	0	1	$900/1 = 900$
Net evaluation row			80	100	0	0	0	
$1800 - 720 \times 4/2 = 360$			$900 - 720 \times 1/2 = 540$					
$5 - 1 \times 2 = 3$			$3 - 1 \times \frac{1}{2} = 5/2$					
$4 - 2 \times 2 = 0$			$1 - 2 \times 1/2 = 0$					
$0 - 1 \times 2 = -2$			$0 - 1 \times 1/2 = -1/2$					
$1 - 0 \times 2 = 1$			$0 - 0 \times 1/2 = 0$					
$0 - 0 \times 2 = 0$			$1 - 0 \times 1/2 = 1$					

Table 2:

			80	100	0	0	0	
Program	Profit/unit	Qty.	X	Y	S ₁	S ₂	S ₃	
Y	100	360	$\frac{1}{2}$	1	$\frac{1}{2}$	0	0	$360 \div 1/2 = 720$
S ₂	0	360	3	0	-2	1	0	$360 \div 3 = 120$
S ₃	0	540	$5/2$	0	-1/2	0	1	$540 \div 5/2 = 216$
Net evaluation row			30	0	-50	0	0	
$360 - 360 \times 1/6 = 300$			$540 - 360 \times 5/6 = 240$					
$\frac{1}{2} - 3 \times 1/6 = 0$			$5/2 - 3 \times 5/6 = 0$					
$1 - 0 \times 1/6 = 1$			$0 - 0 \times 5/6 = 0$					
$\frac{1}{2} - -2 \times 1/6 = 5/6$			$-1/2 - -2 \times 5/6 = 7/6$					
$0 - 1 \times 1/6 = -1/6$			$0 - 1 \times 5/6 = -5/6$					
$0 - 0 \times 1/6 = 0$			$1 - 0 \times 5/6 = 1$					

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Table 3:

			80	100	0	0	0
Program	Profit/unit	Qty.	X	Y	S ₁	S ₂	S ₃
Y	100	300	0	1	5/6	-1/6	0
X	80	120	1	0	-2/3	1/3	0
S ₃	0	240	0	0	7/6	-5/6	1
Net evaluation row			0	0	-500/6	+100/6	
					+160/3	-80/3	0
					$= \frac{180}{6}$	$= -\frac{60}{6}$	

All the values of the net evaluation row of Table 3 are either zero or negative, the optimal program has been obtained.

Here $X = 120$, $y = 300$ and the maximum profit
 $= 80 \times 120 + 100 \times 300 = 9600 + 30,000$
 $= \text{Rs. } 39,600.$

Question 6

Three grades of coal A, B and C contains phosphorus and ash as impurities. In a particular industrial process, fuel up to 100 ton (maximum) is required which could contain ash not more than 3% and phosphorus not more than .03%. It is desired to maximize the profit while satisfying these conditions. There is an unlimited supply of each grade. The percentage of impurities and the profits of each grade are as follows:

Coal	Phosphorus (%)	Ash (%)	Profit in Rs. (per ton)
A	.02	3.0	12.00
B	.04	2.0	15.00
C	.03	5.0	14.00

You are required to formulate the Linear-programming (LP) model to solve it by using simplex method to determine optimal product mix and profit.

Answer

Let X_1 , X_2 and X_3 respectively be the amounts in tons of grades A, B, and C used. The constraints are:

- (i) Phosphorus content must not exceed 0.03%

$$.02 X_1 + .04 X_2 + 0.3 X_3 \leq .03 (X_1 + X_2 + X_3)$$

$$2X_1 + 4 X_2 + 3X_3 \leq 3 (X_1 + X_2 + X_3) \text{ or } -X_1 + X_2 \leq 0$$

- (ii) Ash content must not exceed 3%

$$3X_1 + 2 X_2 + 5 X_3 \leq 3 (X_1 + X_2 + X_3) \text{ or } -X_2 + 2X_3 \leq 0$$

- (iii) Total quantity of fuel required is not more than 100 tons. $X_1 + X_2 + X_3 \leq 100$

The Mathematical formulation of the problem is

$$\text{Maximize } Z = 12 X_1 + 15X_2 + 14 X_3$$

Subject to the constraints:

$$-X_1 + X_2 \leq 0$$

$$-X_2 + X_3 \leq 0$$

$$X_1 + X_2 + X_3 \leq 100$$

$$X_1, X_2, X_3 > 0$$

Introducing slack variable $X_4 > 0, X_5 > 0, X_6 > 0$

			12	15	14	0	0	0
C_b	Y_b	X_b	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6
0	Y_4	0	-1	1*	0	1	0	0
0	Y_5	0	0	-1	2	0	1	0
0	Y_6	100	1	1	1	0	0	1
	Z		-12	-15	-14	0	0	0
C_b	Y_b	X_b	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6
15	Y_2	0	-1	1	0	1	0	0
0	Y_5	0	-1	0	2	1	1	0
0	Y_6	100	2*	0	1	-1	0	1
	Z		-27		-14	15	0	0
C_b	Y_b	X_b	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6

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15	Y_2	50	0	1	$1/2$	$1/2$	0	$1/2$
0	Y_5	50	0	0	$5/2^*$	$1/2$	1	$1/2$
12	Y_1	50	1	0	$1/2$	$-1/2$	0	$1/2$
	Z		0	0	$-1/2$	$3/2$	0	$27/2$
C_b	Y_b	X_b	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6
15	Y_2	40	0	1	0	$2/5$	$-1/5$	$2/5$
14	Y_3	20	0	0	1	$1/5$	$2/5$	$1/5$
12	Y_1	40	1	0	0	$-3/5$	$-1/5$	$2/5$
	Z		0	0	0	$8/5$	$1/5$	$68/5$

The optimum solution is $X_1 = 40$, $X_2 = 40$ and $X_3 = 20$ with maximum $Z = 1360$.

Question 7

What are the practical applications of Linear programming?

Answer

Linear programming can be used to find optional solutions under constraints.

In production:

- pdt. mix under capacity constraints to minimise costs/maximise profits along with marginal costing.
- Inventory management to minimise holding cost, warehousing / transporting from factories to warehouses etc.

Sensitivity Analysis: By providing a range of feasible solutions to decide on discounts on selling price, decisions to make or buy.

Blending: Optional blending of raw materials under supply constraints.

Finance: Portfolio management, interest/receivables management.

Advertisement mix: In advertising campaign – analogous to pdn. management and pdt. mix.

Assignment of personnel to jobs and resource allocation problems.

However, the validity will depend on the manager's ability to establish a proper linear relationship among variables considered.

Question 8

Transport Ltd. Provides tourist vehicles of 3 types – 20-seater vans, 8-seater big cars and 5-seater small cars. These seating capacities are excluding the drivers. The company has 4 vehicles of the 20-seater van type, 10 vehicles of the 8-seater big car types and 20 vehicles of the 5-seater small car types. These vehicles have to be used to transport employees of their client company from their residences to their offices and back. All the residences are in the same housing colony. The offices are at two different places, one is the Head Office and the other is the Branch. Each vehicle plies only one round trip per day, if residence to office in the morning and office to residence in the evening. Each day, 180 officials need to be transported in Route I (from residence to Head Office and back) and 40 officials need to be transported in Route II (from Residence to Branch office and back). The cost per round trip for each type of vehicle along each route is given below.

You are required to formulate the information as a linear programming problem, with the objective of minimising the total cost of hiring vehicles for the client company, subject to the constraints mentioned above. (only formulation is required. Solution is not needed).

Figs. – Rs. /round trip

	20-seater vans	8-seater big cars	5-seater small cars
<i>Route I –</i>			
<i>Residence – Head Office and Back</i>	600	400	300
<i>Route II –</i>			
<i>Residence – Branch Office and Back</i>	500	300	200

Answer

Type	I	II	III	Total no. of passengers
	20 – Seater vans	8 – Seater Big cars	5 – Seater Small cars	
Route I Residence H.O. Residence	600	400	300	180
Route II Residence Br. Residence	500	300	200	40
No. of vehicles	4	10	20	
Max. capacity	80	80	100	220
No. of passengers			260	

Advanced Management Accounting

Let i be the i th route,

and j be the type of vehicle, so that

S_{11} = no. of vans (vehicles on Route I, Type I)

S_{12} = no. of 8 seater cars on Route I

S_{13} = no. of 5 seater cars on Route I

S_{21} = no. of vans — on Route II

S_{22} = no. of 8 seater cars on Route II

S_{23} = no. of 5 seater cars on Route II

Objective:

Minimise

$$\text{Cost } Z = 600 S_{11} + 400 S_{12} + 300 S_{13} + 500 S_{21} + 300 S_{22} + 200 S_{23}$$

Subject to

$$20 S_{11} + 8 S_{12} + 5 S_{13} = 180$$

$$20 S_{21} + 8 S_{22} + 5 S_{23} = 40$$

$$S_{11} + S_{21} \leq 4$$

$$S_{21} + S_{22} \leq 10$$

$$S_{31} + S_{32} \leq 20$$

$$\text{All } s_{ij} \geq 0$$

Question 9

Explain the concept and aim of theory of constraints. What are the key measures of theory of constraints?

Answer

The theory of constraints focuses its attention on constraints and bottlenecks within organisation which hinder speedy production. The main concept is to maximize the rate of manufacturing output is the throughput of the organisation. This requires to examine the bottlenecks and constraints. A bottleneck is an activity within the organization where the demand for that resource is more than its capacity to supply.

A constraint is a situational factor which makes the achievement of objectives / throughput more difficult than it would otherwise, for example of constraint may be lack of skilled labour, lack of customer orders, or the need to achieve high quality in product output.

For example let meeting the customers' delivery schedule be a major constraint in an organisation. The bottleneck may be a certain machine in the factory. Thus bottlenecks and constraints are closely examined to increase throughput.

Key measures of theory of constraints:

- (i) **Throughput contribution:** It is the rate at which the system generates profits through sales. It is defined as, sales less completely variable cost, sales – direct are excluded. Labour costs tend to be partially fixed and conferred are excluded normally.
- (ii) **Investments:** This is the sum of material costs of direct materials, inventory, WIP, finished goods inventory, R & D costs and costs of equipment and buildings.
- (iii) **Other operating costs:** This equals all operating costs (other than direct materials) incurred to earn throughput contribution. Other operating costs include salaries and wages, rent, utilities and depreciation.

Question 10

The costs and selling prices per unit of two products manufacturing by a company are as under:

Product	A (Rs.)	B (Rs.)
Selling Price	500	450
Variable costs:		
Direct Materials @ Rs. 25 per kg.	100	100
Direct Labour @ Rs. 20 per hour	80	40
Painting @ Rs. 30 per hour	30	60
Variable overheads	190	175
Fixed costs @ Rs. 17.50/D.L.Hr.	70	35
Total costs	470	410
Profit	30	40

In any month the maximum availability of inputs is limited to the following:

Direct Materials	480 kg.
Direct Labour hours	400 hours
Painting hours	200 hours

Advanced Management Accounting

Required:

- (i) Formulate a linear programme to determine the production plan which maximizes the profits by using graphical approach.
- (ii) State the optimal product mix and the monthly profit derived from your solution in (i) above.
- (iii) If the company can sell the painting time at Rs. 40 per hour as a separate service, show what modification will be required in the formulation of the linear programming problem. You are required to re-formulate the problem but not to solve.

Answer

Contribution analysis:

Products	A	B
	(Rs.)	(Rs.)
Selling price (A)	<u>500</u>	<u>450</u>
Variable costs:		
Direct Materials	100	100
Direct Labour	80	40
Painting	30	60
Variable Overheads	<u>190</u>	<u>175</u>
Total variable costs (B)	<u>400</u>	<u>375</u>
Contribution (A – B)	<u>100</u>	<u>75</u>
Direct Material per unit	100/25 = 4 kg.	100/25 = 4 kg.
Direct Labour hour per unit	80/20 = 4 hours	40/20 = 2 hours
Painting hour per unit	30/30 = 1 hour	60/30 = 2 hours

Let A be the units to be produced of product A and B be the units to be produced of product B.

LP Problem formulation:

Z Max 100A + 75B

Maximisation of contribution

Subject to:

$$4A + 4B \leq 480$$

Raw material constraint

$$4A + 2B \leq 400$$

Direct Labour hour constraint

$$A + 2B \leq 200$$

Painting hour constraint

$$A, B \geq 0$$

Non negativity constraint

Raw Material Constraint : Put $B = 0$, $A = 120$

Put $A = 0$, $B = 120$

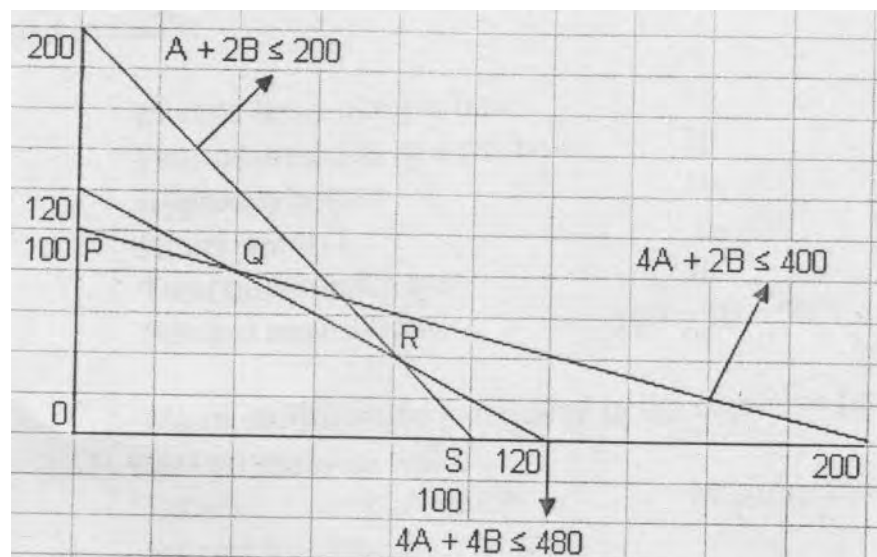
Direct Labour Constraint : Put $B = 0$, $A = 100$

Put $A = 0$, $B = 200$

Painting Constraint : Put $B = 0$, $A = 200$

Put $A = 0$, $B = 100$

The graphical representation will be as under:



$$Q \text{ Intersects } 4A + 2B = 400 \quad (1)$$

$$\text{and } 4A + 4B = 480 \quad (2)$$

Subtracting (2) from (1), we get $-2B = -80$

$$\Rightarrow B = 80/2 = 40$$

Putting value of B in (1), we get $4A + 2 \times 40 = 400$

$$\Rightarrow A = \frac{400 - 80}{4} = 80$$

Advanced Management Accounting

$$R \text{ Intersects } 4A + 4B = 480 \quad (3)$$

$$\text{and } A + 2B = 200 \quad (4)$$

Multiplying (4) by (2) and then subtracting from (3), we get

$$2A = 80$$

$$\Rightarrow A = 40$$

Putting the value of A in (4), we get $2B = 200 - 40$

$$\Rightarrow B = 80.$$

Evaluation of corner points:

Point	Products		Contribution		Total Contribution
	A	B	A (Rs.) 100 per unit	B (Rs.) 75 per unit	Rs.
P	0	100	0	7,500	7,500
Q	80	40	8,000	3,000	11,000
R	40	80	4,000	6,000	10,000
S	100	0	10,000	0	10,000

Optimal product mix is Q

Product	Units	Contribution Rs.
A	80	8,000
B	40	3,000
Total contribution		11,000
Less: Fixed costs 400 D.L. Hrs. $\times$ Rs. 17.50		7,000
Optimal Profit		4,000

(iii) If the painting time can be sold at Rs. 40 per hour the opportunity cost is calculated as under:

	A (Rs.)	B (Rs.)
Income from sale per hour	40	40

Linear Programming

Painting variable cost per hour	30	30
Opportunity cost	10	10
Painting hours per unit	1	2
Opportunity cost	10	20
Revised contribution	$100 - 10 = 90$	$75 - 20 = 55$

Hence, modification is required in the objective function.

Re-formulated problem will be:

Z Max. $90A + 55B$ Maximisation of contribution

Subject to:

$4A + 4B \leq 480$	Raw Material constraint
$4A + 2B \leq 400$	Direct Labour hour constraint
$A + 2B \leq 200$	Painting hour constraint
$A, B \geq 0$	Non-negativity constraint

Question 11

The following matrix gives the unit cost of transporting a product from production plants P_1 , P_2 and P_3 to destinations D_1 , D_2 and D_3 . Plants P_1 , P_2 and P_3 have a maximum production of 65, 24 and 111 units respectively and destinations D_1 , D_2 and D_3 must receive at least 60, 65 and 75 units respectively:

To From	D_1	D_2	D_3	Supply
P_1	400	600	800	65
P_2	1,000	1,200	1,400	24
P_3	500	900	700	111
Demand	60	65	75	200

You are required to formulate the above as a linear programming problem. (Only formulation is needed. Please do not solve).

Answer

Let $p_i d_j$ be the variable to denote the number of units of product from the i th plant to the j th destination, so that

$P_1 d_1$ = transport from plant P_1 to D_1

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P_2d_2 = transport from plant P_2 to D_2 etc.

Objective function

Minimize $z = 400 p_1d_1 + 600 p_1d_2 + 800 p_1d_3 + 1000 p_2d_1 + 1200 p_2d_2 + 1400 p_2d_3 + 500 p_3d_1 + 900 p_3d_2 + 700 p_3d_3$.

Subject to:

$$\left. \begin{array}{l} p_1d_1 + p_1d_2 + p_1d_3 \leq 65 \\ p_2d_1 + p_2d_2 + p_2d_3 \leq 24 \\ p_3d_1 + p_3d_2 + p_3d_3 \leq 111 \end{array} \right\} \text{(Plant constraints)}$$

and

$$\left. \begin{array}{l} p_1d_1 + p_2d_1 + p_3d_1 \geq 60 \\ p_1d_2 + p_2d_2 + p_3d_2 \geq 65 \\ p_1d_3 + p_2d_3 + p_3d_3 \geq 75 \end{array} \right\} \text{(destination constraints)}$$

all $p_id_j \geq 0$

Question 12

Formulate the dual for the following linear program: (6 Marks)

Maximise: $100x_1 + 90x_2 + 40x_3 + 60x_4$

Subject to

$$6x_1 + 4x_2 + 8x_3 + 4x_4 \leq 140$$

$$10x_1 + 10x_2 + 2x_3 + 6x_4 \leq 120$$

$$10x_1 + 12x_2 + 6x_3 + 2x_4 \leq 50$$

$$x_1, x_2, x_3, x_4 \geq 0$$

(Only formulation is required. Please do not solve.)

Answer

Dual:

$$\text{Minimise} \quad 140u_1 + 120u_2 + 50u_3$$

$$\text{S.T.} \quad 6u_1 + 10u_2 + 10u_3 \geq 100$$

$$4u_1 + 10u_2 + 12u_3 \geq 90$$

$$8u_1 + 2u_2 + 6u_3 \geq 40$$

$$4u_1 + 6u_2 + 2u_3 \geq 60$$

$$u_1, u_2, u_3, u_4 \geq 0$$

Question 13

The following is a linear programming problem. You are required to set up the initial simplex tableau. (Please do not attempt further iterations or solution):

Maximise

$$100x_1 + 80x_2$$

Subject to

$$3x_1 + 5x_2 \leq 150$$

$$x_2 \leq 20$$

$$8x_1 + 5x_2 \leq 300$$

$$x_1 + x_2 \geq 25$$

$$x_1, x_2 \geq 0$$

Answer

Under the usual notations where

S1, S2, S3 are stock Variables,

A4 = the artificial variable

S4 = Surplus Variable

We have,

$$\text{Max. } Z = 100x_1 + 80x_2 + 0S_1 + 0S_2 + 0S_3 + 0S_4 - M A_4.$$

S.t.

$$3x_1 + 5x_2 + S_1 = 150$$

$$x_2 + S_2 = 20$$

$$8x_1 + 5x_2 + S_3 = 300$$

$$x_1 + x_2 - S_4 + A_4 = 25$$

		x_1	x_2	S_1	S_2	S_3	S_4	A_4		
Basis	C_j	100	80	0	0	0	0	- M		
	C_B									

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S ₁	0	3	5	1	0	0	0	0	150	√
S ₂	0	0	1	0	1	0	0	0	20	√
S ₃	0	8	5	0	0	1	0	0	300	√
A ₄	- M	1	1	0	0	0	-1	1	25	√
Z _j		- M	- M	0	0	0	M	-M	-25M	√
C _j -Z _j		100+M	80+M	0	0	0	-M	0		√

Question 14

An oil refinery can blend three grades of crude oil to produce quality A and quality B petrol. Two possible blending processes are available. For each production run, the older process uses 5 units of crude Q, 7 units of crude P and 2 units of crude R and produces 9 units of A and 7 units of B. The newer process uses 3 units of crude Q, 9 unit of crude P and 4 units of crude R to produce 5 units of A and 9 units of B.

Because of prior contract commitments, the refinery must produce at least 500 units of A and at lease 300 units of B for the next month. It has ,1,500 units of crude Q, 1,900 units of crude P and 1,000 of crude R. For each unit of A, refinery receives Rs.60 while for each unit of B, it receives Rs.90

Formulate the problem as linear programming model so as to maximize the revenue.

Answer

$$\begin{aligned}\text{Maximize } Z &= 60 (9x_1 + 5x_2) + 90 (7x_1 + 9x_2) \\ &= 1170x_1 + 1110x_2\end{aligned}$$

Subject to $9x_1 + 5x_2 \geq 500$ commitment for A
 $7x_1 + 9x_2 \geq 300$ commitment for B
 $5x_1 + 3x_2 \leq 1500$ availability of Q
 $7x_1 + 9x_2 \leq 1900$ availability of P
 $2x_1 + 4x_2 \leq 1000$ availability of R
and $x_1 \geq 0, x_2 \geq 0$.

Question 15

Write short notes on the characteristics of the dual problem.

Answer

Characteristics of the dual problem:

1. For any linear programming model called primal model, there exists a companion model called the dual model.
2. The number of constraints in the primal model equals the number of variables in the dual model.
3. The number of variables in the primal problem equals the number of constraints in the dual model.
4. If the primal model is a maximization problem then the dual model will be of the form less than or equal to, " $\leq$ " while the restrictions in the dual problem will be of the form-greater than or equal to, " $\geq$ ".
5. The solution of the primal model yields the solution of the dual model. Also, an optimal simplex table for the dual model yields the optimal solution to the primal model. Further, the objective functions of the two optimal tables will have identical values.
6. Dual of the primal's dual problem is the primal problem itself.
7. Feasible solutions to a primal and dual problem are both optimal if the complementary slackness conditions hold, that is, (value of a primal variable) $\times$ (value of the corresponding dual surplus variable) = 0 or (value of a primal slack variable) $\times$ (value of the corresponding dual variable) = 0.
If this relationship does not hold, then either the primal solution or the dual solution or both are not optimal.
8. If the primal problem has no optimal solution because of infeasibility, then the dual problem will have no optimal solution because of unboundedness.
9. If the primal has no optimal solution because of unboundedness, then the dual will have no optimal solution because of infeasibility.

EXERCISE**Question 1**

A Sports Club is engaged in the development of their players by feeding them certain minimum amount of Vitamins (say A, B and C), in addition to their normal diet. In view of this, two types of products X and Y are purchased from the market. The contents of Vitamin constituents per unit, are shown in the following table:

Vitamin Constituents	Vitamin contents in products		Minimum requirement for each player
	X	Y	
A	36	06	108
B	03	12	36
C	20	10	100

The cost of product X is Rs.20 and that of Y is Rs.40.

Formulate the linear programming problem for the above and minimize the total cost, and solve problem by using graphic method.

Answer

The optimal solution is to purchase 4 units of product X and 2 units of product Y in order to maintain a minimum cost of Rs.160/-.

Question 2

A manufacturer produces three products Y_1 , Y_2 , Y_3 from three raw materials X_1 , X_2 , X_3 . The cost of raw materials X_1 , X_2 and X_3 is Rs.30, Rs.50 and Rs.120 per kg respectively and they are available in a limited quantity viz 20 kg of X_1 , 15 kg of X_2 and 10 kg of X_3 . The selling price of Y_1 , Y_2 and Y_3 is Rs.90, Rs.100 and Rs.120 per kg respectively. In order to produce 1 kg of Y_1 , $\frac{1}{2}$ kg of X_1 , $\frac{1}{4}$ kg of X_2 and $\frac{1}{4}$ kg of X_3 are required. Similarly to produce 1 kg of Y_2 , $\frac{3}{7}$ kg of X_1 , $\frac{2}{7}$ kg of X_2 and $\frac{2}{7}$ kg of X_3 and to produce 1 kg Y_3 , $\frac{2}{3}$ kg of X_2 and $\frac{1}{3}$ kg of X_3 will be required.

Formulate the linear programming problem to maximize the profit.

Answer

$$\text{Maximise } Z = 32.50 y_1 + 38.57 y_2 + 46.67 y_3$$

$$\frac{1}{2} y_1 + \frac{3}{7} y_2 \leq 20 \text{ or } 7 y_1 + 6 y_2 \leq 280$$

$$\frac{1}{4} y_1 + \frac{2}{7} y_2 + \frac{2}{3} y_3 \leq 15 \text{ or } 21 y_1 + 24 y_2 + 56 y_3 \leq 1,260$$

$$\frac{1}{4} y_1 + \frac{2}{7} y_2 + \frac{1}{3} y_3 \leq 10 \text{ or } 21 y_1 + 24 y_2 + 28 y_3 \leq 840$$

where Y_1, Y_2 and $Y_3 \geq 0$

Question 3

Write short notes on applications and limitation of Linear Programming Techniques.

Answer

Refer to Chapter 11: Paragraph: 11.9

Question 4

In a chemical industry two products A and B are made involving two operations. The production of B also results in a by-product C. The product A can be sold at a profit of Rs.3 per unit and B at a profit of Rs.8 per unit. The by-product C has a profit of Rs.2 per unit. Forecast show that upto 5 units of C can be sold. The company gets 3 units of C for each unit of B produced. The manufacturing times are 3 h per unit and on each of the operation one and two and 4 h and 5 h per unit for B on operation one and two respectively. Because the product C results from producing B, no time is used in producing C. The available times are 18 h and 21 h of operation one and two respectively. The company desires to know that how much A and B should be produced keeping c in mind to make the highest profit. Formulate LP model for this problem.

Answer

Maximise $Z = 3x_1 + 8x_2 + 2x_3$

Subject to the constraints

$$3x_1 + 4x_2 \leq 18$$

$$3x_1 + 5x_2 \leq 21$$

$$x_3 \leq 5, x_3 = 3x_2$$

$$x_1, x_2, x_3 \geq 0$$

Question 5

An advertising firm desires to reach two types of audiences – customers with annual income of more than Rs.40,000 (target audience A) and customers with annual income of less than Rs.40,000 (target audience B). The total advertising budget is Rs.2,00,000. One programme of T.V. advertising costs Rs.50,000 and one programme of Radio advertising costs Rs.20,000. Contract conditions ordinarily require that there should be at least 3 programmes on T.V. and the number of programmes on Radio must not exceed 5. Survey indicates that a single T.V. programme reaches 7,50,000 customers in target audience A and 1,50,000 in target audience B. One Radio programme reaches 40,000 customers in target audience A and 2,60,000 in target audience B.

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Formulate this as a linear programming problem and determine the media mix to maximize the total reach using graphic method.

Answer

the advertising firm should give 4 programmes on TV and no programme on Radio in order to achieve a maximum reach of 36,00,000 customers.

Question 6

Let us assume that you have inherited Rs.1,00,000 from your father-in-law that can be invested in a combination of only two stock portfolios, with the maximum investment allowed in either portfolio set at Rs.75,000. The first portfolio has an average rate of return of 10%, whereas the second has 20%. In terms of risk factors associated with these portfolios, the first has a risk rating of 4 (on a scale from 0 to 10), and the second has 9. Since you wish to maximize your return, you will not accept an average rate of return below 12% or a risk factor above 6. Hence, you then face the important question. How much should you invest in each portfolio?

Formulate this as a Linear Programming Problem and solve it by Graphic Method.

Answer

the company should invest Rs.60,000 in first portfolio and Rs.40,000 in second portfolio to achieve the maximum average rate of return of Rs.14,000.

Question 7

A firm buys casting of P and Q type of parts and sells them as finished product after machining, boring and polishing. The purchasing cost for casting are Rs.3 and Rs.4 each for parts P and Q and selling costs are Rs.8 and Rs.10 respectively. The per hour capacity of machines used for machining, boring and polishing for two products is given below:

Capacity (per hour)	Parts	
	P	C
Machining	30	50
Boring	30	45
Polishing	45	30

The running costs for machining, boring and polishing are Rs.30, Rs.22.5 and Rs.22.5 per hour respectively.

Formulate the linear programming problem to find out the product mix to maximize the profit.

Answer

$$\text{Maximise } Z = 2.75x + 4.15y$$

Subject to the constraints

$$50x + 30y \leq 1,500$$

$$45x + 30y \leq 1,350$$

$$30x + 45y \leq 1,350$$

$$\text{where } x, y \geq 0$$

Question 8

A Mutual Fund Company has Rs.20 lakhs available for investment in Government Bonds, blue chip stocks, speculative stocks and short-term bank deposits. The annual expected return and risk factor are given below:

Type of investment	Annual Expected return (%)	Risk Factor (0 to 100)
Government Bonds	14	12
Blue Chip Stocks	19	24
Speculative Stocks	23	48
Short term deposits	12	6

Mutual fund is required to keep at least Rs.2 lakhs in short-term deposits and not to exceed an average risk factor of 42. Speculative stocks must be at most 20 percent of the total amount invested. How should mutual fund invest the funds so as to maximize its total expected annual return? Formulate this as a Linear Programming Problem. Do not solve it.

Answer

Objective function:

$$\text{Maximise } Z = 0.14x_1 + 0.19x_2 + 0.23x_3 + 0.12x_4$$

Subject to the constraints:

$$x_1 + x_2 + x_3 + x_4 \leq 20,00,000$$

$$x_4 \geq 2,00,000$$

$$-30x_1 - 18x_2 + 6x_3 - 36x_4 \leq 0$$

$$-0.2x_1 - 0.2x_2 + 0.8x_3 + 0.2x_4 \leq 0$$

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Where $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$ and $x_4 \geq 0$

Question 9

The owner of Fancy Goods Shop is interested to determine, how many advertisements to release in the selected three magazines A, B and C. His main purpose is to advertise in such a way that total exposure to principal buyers of his goods is maximized. Percentages of readers for each magazine are known. Exposure in any particular magazine is the number of advertisements released multiplied by the number of principal buyers. The following data are available:

Particulars	Magazines		
	A	B	C
Readers	1.0 Lakhs	0.6 Lakhs	0.4 Lakhs
Principal buyers	20%	15%	8%
Cost per advertisement	8,000	6,000	5,000

The budgeted amount is at the most Rs.1.0 lakh for the advertisements. The owner has already decided that magazine A should have no more than 15 advertisements and that B and C each gets at least 8 advertisements. Formulate a Linear Programming model for this problem.

Answer

$$\text{Maximise } Z = 20,000 x_1 + 9,000 x_2 + 3,200 x_3$$

$$\text{subject to } 8,000 x_1 + 6,000 x_2 + 5,000 x_3 \leq 1,00,000$$

$$x_1 \leq 15, x_2 \geq 8,$$

$$\text{where } x_1, x_2 \text{ and } x_3 \geq 0$$

Question 10

An agriculturist has a farm with 125 acres. He produces Radish, Mutter and Potato. Whatever he raises is fully sold in the market. He gets Rs.5 for Radish per kg Rs.4 for Mutter per kg and Rs. for Potato per kg. The average yield is 1,500 kg of Radish per acre, 1,800 kg of Mutter per acre and 1,200 kg of Potato per acre. To produce each 100 kg of Radish and Mutter and to produce each 80 kg of Potato, a sum of Rs.12.50 has to be used for manure. Labour required for each acre to raise the crop is 6 man days for Radish and Potato each and 5 man days for Mutter. A total of 500 man days of labour at a rate of Rs.40 per man day are available.

Formulate this as a Linear Programming model to maximize the Agriculturist's total profit.

Answer

$$\text{Maximise } Z = 7,072.5x_1 + 6,775x_2 + 5572.5x_3$$

Subject to following constraints:

$$x_1 + x_2 + x_3 \leq 125$$

$$6x_1 + 5x_2 + 6x_3 \leq 500$$

$$\text{Where } x_1, x_2 \text{ and } x_3 \geq 0$$

Question 11

A firm produces three products A, B and C. It uses two types of raw materials I and II of which 5,000 and 7,500 units respectively are available. The raw material requirements per unit of the products are given below:

Raw Material	Requirement per unit of Product		
	A	B	C
I	3	4	5
II	5	3	5

The labour time for each unit of product A is twice that of product B and three times that of product C. The entire labour force of the firm can produce the equivalent of 3,000 units. The minimum demand of the three products is 600, 650 and 500 units respectively. Also the ratios of the number of units produced must be equal to 2: 3: 4. Assuming the profits per unit of A, B and C as Rs.50, 50 and 80 respectively.

Formulate the problem as a linear programming model in order to determine the number of units of each product, which will maximize the profit.

Answer

$$\text{Maximise } Z = 50x_1 + 50x_2 + 80x_3$$

Subject to the constraints

$$3x_1 + 4x_2 + 5x_3 \leq 5,000$$

$$5x_1 + 3x_2 + 5x_3 \leq 7,500$$

$$6x_1 + 3x_3 + 2x_3 \leq 18,000$$

$$3x_1 = 2x_2 \text{ and } 4x_2 = 3x_3$$

$$x_1 \geq 600, x_2 \geq 650 \text{ and } x_3 \geq 500$$